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SEMIEMPIRICAL MODEL OF IMPACT INTERACTION OF A DISPERSE IMPURITY PARTICLE WITH A SURFACE IN A GAS SUSPENSION FLOW

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A mathematical model describing the dynamics of impact of a spherical particle on a solid surface is proposed and investigated. In closing the model, use is made of the experimental mean statistical values of the coefficients of restitution of the components of the velocity vector of the center of mass of the particle normal and tangential to the surface. The model permits a physically correct description of particle rotation upon impact and determination of its angular rotational velocity.

When a gas suspension flows past bodies or obstacles, solid impurity particles recoil, as a rule, from the surface, thus making the flow picture of the impurity phase more complicated. In this case, in order to describe adequately the dynamics of particles in the disturbed region, it is necessary to determine correctly their parameters immediately after impact on a surface. Since real particles often possess an irregular shape, the parameters of their recoil have a random nature and may be evaluated only in some mean statistical sense, which is especially emphasized in [1–3]. Even if a particle is not destroyed upon impact and surface erosion is insignificant, the particle-surface interaction is a very complicated process that depends on the physicomechanical properties of their materials, the temperatures, the particle size, the magnitude and orientation of its velocity vector before impact, and so on [4]. The most reliable results on determination of the particle parameters at the moment of recoil are obtained experimentally and pertain to the coefficients of restitution of the components a_n and a_τ of the velocity vector of the center of mass of a mean statistical particle normal and tangential to the surface [1–3].

In collision theory discussed in theoretical mechanics (see, e.g., [5]), the interaction of a particle with a surface is considered as instantaneous, point contact and shear forces are postulated to be absent. This results in the fact that the tangential velocity of the center of mass of the particle does not change upon impact, i.e., there the equality $a_\tau = 1$ is fulfilled. The change in the normal component of the velocity of the center of mass of the particle upon impact is calculated in the classical theory based on the Newton hypothesis that the coefficient of restitution normal component of the velocity of the point of contact of a particle with a surface ε is a physical constant for given materials of the particle and the surface and is independent of the impact velocity, the angle of incidence, and the particle size. For a spherical particle, the equality $a_n = \varepsilon$ is fulfilled. Although the assumption $a_\tau = 1$ and $a_n = \text{const}$ (≤ 1) sharply contradicts experimental data [1–3], precisely these equalities are used to calculate the recoil of spherical particles from a surface in the overwhelming majority of works devoted to modeling flows in the two-phase aerodynamics problems [6–9]. Apparently, this is associated with the absence of a more rigorous reliable theory of collision.

In [10], the experimental coefficients a_n and a_τ obtained in [1] are used to determine the velocity vector of particles immediately after impact. However, the authors of [10] do not take into consideration the fact that the action of tangential momentum on a particle upon impact leads not only to a decrease in the tangential velocity component of the particle but also to its rotation. As a result, recoiling particles are treated in [10] as nonrotating

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and, consequently, the Magnus force is not considered. At the same time, tentative calculations for rotating particles* show that the Magnus force may change the picture of an impurity flow substantially.

The angular velocity of particle rotation just after impact may be determined correctly from the equations of change in momentum and angular momentum if the coefficient of restitution of the tangential component of the velocity vector not of the center of mass of the particle but of the point of contact of it with the surface is known [11]. However, it is rather difficult to determine experimentally the above coefficients for the point of contact. Application of a similar approach to the calculation of the angular velocity of a particle, proceeding from known experimental a_τ values for its center of mass, yields a physically incorrect result: a particle may be overrotated immediately after impact. Let us write the equations for the change in the component of momentum and angular momentum tangential to a surface during impact time of an initially nonrotating spherical particle:

$$mu_1 (a_\tau - 1) = - F_\tau, \quad J\omega_2 = - F_\tau r.$$

From these equations with account for the relation for the moment of inertia of a particle $J = 2mr^2/5$ we find

$$\omega_2 = \frac{5}{2} \frac{u_1}{r} (a_\tau - 1).$$

Expressing the velocity of the point of contact of a particle just after impact $u_{c2} = u_2 + \omega_2 r$ in terms of known quantities, we arrive at

$$u_{c2} = \frac{7}{2} u_1 \left(a_\tau - \frac{5}{7} \right).$$

Hence it is seen that at $a_\tau < 5/7$ the quantity u_{c2} becomes negative, i.e., the velocity vector of the point of contact of a particle relative to the surface at the moment of its recoil is directed opposite to the velocity vector of the center of mass, which contradicts physical concepts. Since experimental a_τ values for isometric particles in wide ranges of impact conditions may be substantially less than 5/7 [1–3], the above model of ω_2 calculations is obviously inapplicable under these conditions.

Thus, currently neither an adequate deductive theory of impact of a disperse particle on a surface nor a satisfactory approximate model based on reliable experimental data is available.

In the present work a semiempirical model of impact interaction of an impurity particle with a solid surface in a flow is proposed and examined. The model is based on the laws of mechanics, some realistic physical assumptions, and experimental data on the coefficients of restitution of the components of the velocity vector of the center of mass of the particle. This model allows determination of the velocity of particle rotation at the moment of its recoil from the surface without leading to the paradox described above.

We now consider the recoil of some mean-statistical disperse impurity particle from the surface of a solid or obstacle. We assume that its shape is close to spherical, and from the viewpoint of the dynamic and geometric characteristics used below, it may be treated approximately as a sphere. This assumption is obviously valid for powder materials subjected to special technological spheroidization. It is also acceptable for isometric particles of irregular shape [12]. But this assumption is not fulfilled for particles in the form of platelets or fibers and therefore we do not investigate such particles in the present work. The impact of a spherical particle on a surface is considered to be two-dimensional (the velocity vectors of the center of mass of the particle before and after impact lie in a single plane with the vector of the normal to the surface at the point of contact at the moment of impact). In this case, the velocity vector of the center of mass of the particle may be described completely by just two components

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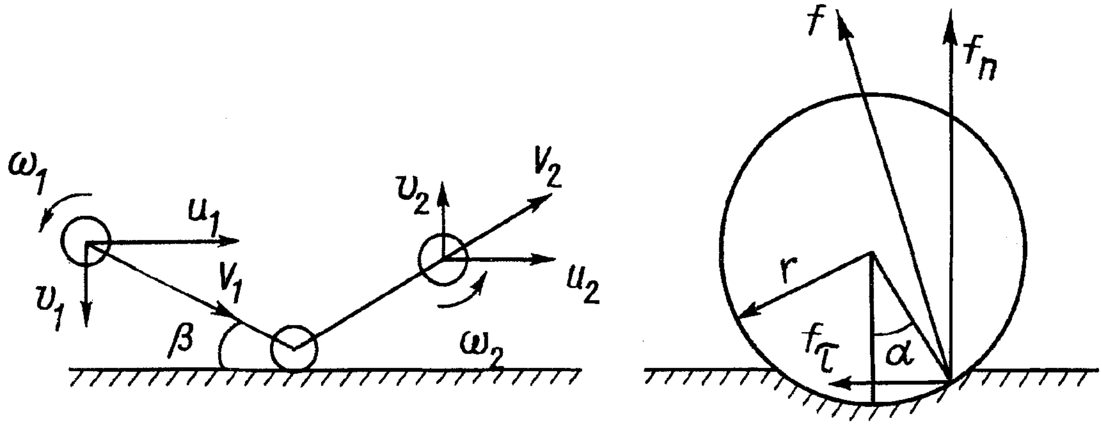

Fig. 1. Schematic of the impact of a particle on an obstacle and notation.

lying in the collision plane, and the angular velocity vector by just the component perpendicular to the collision plane, which simplifies the problem substantially. We also assume that the particle is not deformed during impact but the surface of the solid or the obstacle may be deformed. Such an assumption is valid when the hardness of the particle material substantially exceeds that of the obstacle material (for instance, in impact of electrocorundum particles on an unhardened steel or copper plate). Thus, we deal with the impact interaction of an underformable spherical particle with an elastoplastic obstacle (see Fig. 1).

Unlike the classical theory, we assume that the spot of contact between a particle and an obstacle during impact has finite dimensions. If the velocity vector of an incident particle is at an acute angle to the surface ($\beta < \pi/2$), it should be expected that the profile of stresses developing in the obstacle at the particle-obstacle interface and applied to the particle is asymmetric. Then the vector of the resultant of these stresses f is at some nonzero angle to the normal vector n, and the point of application of f is displaced from the center of the contact spot in the direction of u_1. This displacement may be specified uniquely by the angle α (Fig. 1). As is seen, the normal component f_n creates an angular momentum relatively to the particle's center, which must be taken into account, along with the moment of the force f_τ, in the equation of change in the angular momentum of the particle upon impact. (In the classical theory the angular momentum due to the force f_n for a spherical particle is equal to zero.)

The equations of change in the tangential and normal momenta of the particle in the collision plane and the equation of change in the angular momentum of the particle for the adopted scheme of its force interaction with the obstacle are as follows:

$$\frac{du}{dt} = \frac{f_\tau}{m}, \quad \frac{dv}{dt} = \frac{f_n}{m}, \quad \frac{d\omega}{dt} = \frac{r}{J} (f_n \sin \alpha + f_\tau \cos \alpha). \tag{1}$$

We assume that during impact the force of resistance to particle motion in the tangential direction is proportional to the normal response and the characteristic slip of the particle relative to the obstacle at the contact spot :

$$f_\tau = - \kappa f_n (u + \omega r). \tag{2}$$

Here κ (> 0) means a resistance coefficient that takes into account both the slip fraction and the resistance of the obstacle to deformation of it by a particle in the tangential direction which should be accounted for in the case of surface erosion. The introduced phenomenological dependence (2) is a generalized law of dry friction.

The second equation in (1) may be integrated independently of the others. As a result, we obtain

$$m (v_2 - v_1) = \int_0^T f_n dt \equiv F_n. \tag{3}$$

We now introduce the mean normal force over the impact time using the relation $\hat{f}_n - F_n/T$ and express v_2 in terms of v_1 and the coefficient of restitution a_n: $v_2 = -a_n v_1$. Then equality (3) is transformed to the form

$$\hat{f}_n T = -m v_1 (a_n + 1). \tag{4}$$

In the two remaining equations of system (1) we substitute $\hat{f}_n$ for f_n. Moreover, we assume that α and κ are constant during impact and are independent of the angular velocity ω_1 of the incident particle. Although these assumptions are not rigorous, they are, in our opinion, quite acceptable since the aim of modeling the impact dynamics is determination of particle parameters at the moment of recoil from a surface. With account for relation (2) and the assumptions made, the first and third equations of (1) may be written in the form

$$\frac{du}{dt} = -\frac{\hat{f}_n}{m} \kappa (u + \omega r), \quad \frac{d\omega}{dt} = \frac{\hat{f}_n}{J} r [\sin\alpha - \kappa (u + \omega r) \cos\alpha]. \tag{5}$$

These two equations form a closed linear system in which f_n is prescribed by relation (4) and κ and α are parameters.

The initial conditions for system (5) are as follows:

$$t = 0: \ u = u_1, \ \omega = \omega_1. \tag{6}$$

Cauchy problem (5), (6) has an exact analytical solution that at $t = T$ (at the moment of particle recoil) yields

$$\begin{gathered} u_2 = \frac{1}{2 + A} \left[u_1 (2E + A) + 2\omega_1 r (E - 1) + \frac{2A \tan\alpha}{(2 + A)\kappa} (1 - E + B) \right], \\ \omega_2 = \frac{1}{2 + A} \left[\frac{u_1}{r} A (E - 1) + \omega_1 (AE + 2) + \frac{A^2 \tan\alpha}{(2 + A) r\kappa} \left(1 - E + \frac{2B}{A} \right) \right], \\ A = 5\cos\alpha, \quad B = \kappa v_1 (a_n + 1) \frac{2 + A}{2}, \quad E = \exp B. \end{gathered} \tag{7}$$

In deriving equalities (7) it has been taken into account that the moment of inertia of a spherical particle relative to its center of mass is $J = 2mr^2/5$. Further analysis of solution (7) is concerned with choosing the values of the as yet undetermined parameters κ and α.

All known experimental data on the coefficients a_τ and a_n were obtained for particles that do not rotate before impact, and therefore it is natural to require that u_2 from (7) be equal at $\omega_1 = 0$ to the experimental value of $a_\tau u_1$:

$$\frac{1}{2 + A} \left[u_1 (2E + A) + \frac{2A \tan\alpha}{(2 + A)\kappa} (1 - E + B) \right] = a_\tau u_1. \tag{8}$$

Relation (8) contains, along with α and κ, the quantities u_1, v_1 (entering B), a_τ, and a_n. The components u_1 and v_1 are expressed in terms of the magnitude of the velocity vector of the incident particle V_1 and the angle β in the following way (see Fig. 1): $u_1 = V_1 \cos\beta$, $v_1 = -V_1 \sin\beta$. In an experiment, for prescribed particles and material of the obstacle the dependences $a_\tau(\beta)$ and $a_n(\beta)$ [1, 2] or $a_\tau(\beta)$ and $a_n(V_1, \beta)$ [3] are usually determined. In both cases relation (8) gives the functional dependence $\alpha = \alpha(\kappa, V_1, \beta)$, which for fixed V_1 and β obviously turns into $\alpha(\kappa)$.

In the present work, in all numerical calculations we have used experimental data on the coefficients of restitution a_τ and a_n from [3], in which the interaction of electrocorundum particles with obstacles made of different materials was investigated. Here the dependence $a_n(V_1, \beta)$, approximating experimental points for different obstacles, is taken in the same form as in [3]: $a_n = 1 - [1 - \exp(-0.1 V_1^{0.61})] \sin\beta$. The dependences $a_\tau(\beta)$ for different materials of the obstacle are determined, unlike [3], with account for the additional condition

TABLE 1. Coefficient Values in the Approximate Dependence $a_\tau(\beta)$ for Different Materials of the Obstacle

Obstacle material	Coefficient in formula (9)			
	C_0	C_1	C_2	C_3
Steel	0.690	−0.288	0.114	0.0219
Copper	0.588	−0.354	0.0762	0.0547
Lead	0.430	−0.239	−0.0759	0.108

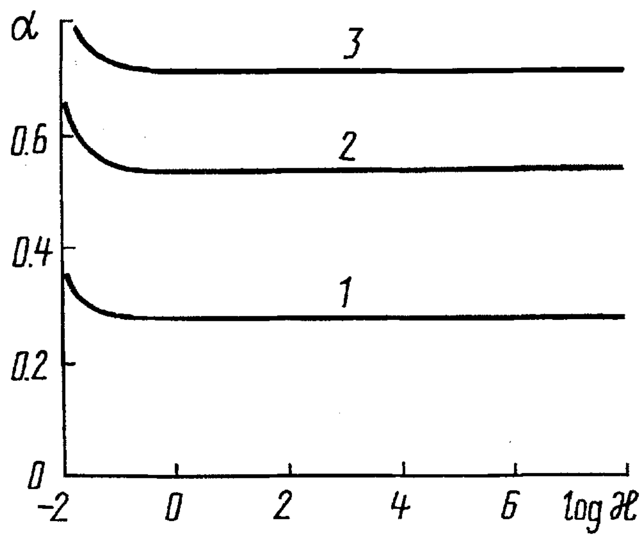

Fig. 2. Angle α (rad) versus coefficient κ (sec/m) for different materials of the obstacle (V_1 = 200 m/sec, $\beta = 25^0$): 1) steel; 2) copper; 3) lead.

$$\beta = \frac{\pi}{2}: \quad \frac{d^k a_\tau}{d\beta^k} = 0 , \quad k = 1 , 3 , \ldots ,$$

which is the symmetry condition for the function a_τ (β) concerning strictly normal impact of a particle on a surface. The experimental points for a_τ from [3] are approximated by the following polynomials:

$$a_\tau = C_0 + C_1 \left(\frac{\pi}{2} - \beta\right)^2 + C_2 \left(\frac{\pi}{2} - \beta\right)^4 + C_3 \left(\frac{\pi}{2} - \beta\right)^6 \tag{9}$$

by the least-squares method* (the point for a copper obstacle at $\beta = 75^0$ disagrees sharply with the general picture [3] and therefore it has not been taken into account in the calculations). The coefficients of (9) for various materials of the obstacle are presented in Table 1.

For the aforementiomed $a_n(V_1, \beta)$ and $a_\tau(\beta)$ we investigated $\alpha(\kappa)$ for different V_1, β, and materials of the obstacle using equality (8). A typical form of these dependences is shown in Fig. 2. The function $\alpha(\kappa)$ decreases with increase in κ and its graph tends rapidly to a horizontal asymptote, virtually attaining it at $\kappa \sim 1$. As calculation results show, α changes rather weakly (by 15–20%) in a wide range of the coefficient κ (for 0.01 sec/m $< \kappa < \infty$). Here ω_2 obtained by (7) also changes insignificantly. As is seen in the figure, with an increase in the plasticity of the obstacle material, the angle α increases. This may be interpreted as a result of deeper penetration of a particle into a more plastic material (for fixed V_1 and β), which corresponds fully to physical concepts.

* The experimentally determined numerical values of the coefficient a_τ for different materials of the obstacle and angles β, represented by points on the curves in [3], have been kindly provided by V. A. Lashkov.

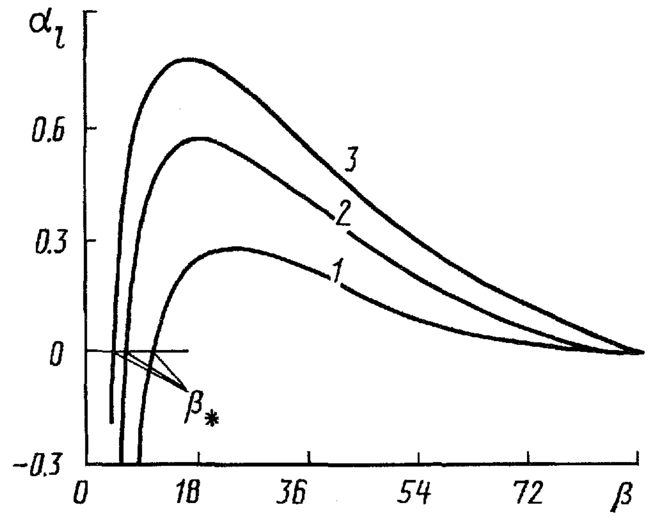

Fig. 3. Limiting angle α_l (rad) versus the angle of incidence β (deg) of the particle for different materials of the obstacle (V_1 = 200 m/sec): 1) steel; 2) copper; 3) lead.

If we assume that the actual value of κ in phenomenological law (2) lies in the aforementioned range, then the angle α may be assumed, with sufficiently high accuracy, equal to the limiting value $\alpha_l = \lim\limits_{\kappa \to \infty} \alpha(\kappa)$. To determine α_l, we pass in (8) to the limit as $\kappa \to \infty$. After simple transformations we arrive at the equation

$$A_l - A_l \tan \alpha_l \tan \beta \, (a_n + 1) = (2 + A_l) \, a_\tau \,, \quad A_l = 5 \cos \alpha_l \,,$$

which reduces to a quadratic one relative to $z = \tan(\alpha_l/2)$:

$$(5 - 3a_\tau) \, z^2 + 10 \tan \beta \, (a_n + 1) \, z + 7a_\tau - 5 = 0 \,. \tag{10}$$

From physical reasoning, of the two roots $z_{1,2}$ we must take z_1, which gives a smaller α_l in absolute magnitude, which corresponds to a lesser penetration of the particle into the obstacle. For a fixed value of V_1 the solution of Eq. (10) and subsequent transition to α_l give $\alpha_l(\beta)$ (see Fig. 3). It is pertinent to note that with decreasing β after passage through β_* Eq. (10) no longer has real roots.

The β_* values in Fig. 3 are obtained at $a_\tau = 5/7$. If $a_\tau < 5/7$, then $\alpha_l > 0$; if $a_\tau > 5/7$, then $\alpha_l < 0$. This result, obtained at $\kappa \to \infty$, makes it possible to give a physical interpretation to the paradox discussed at the beginning of the paper.

Indeed, if the incident particle does not rotate, the highest angular velocity at the moment of recoil from the surface is obtained for rigid (without slip) cohesion of the particle with the obstacle, i.e., at $\kappa \to \infty$. In this case, accounting only for the tangential force f_τ in the equation of change in the angular momentum of the particle results, at $a_\tau < 5/7$, in the fact that ω_2 exceeds the maximum permissible value, i.e., the particle is overrotated. Introduction of a nonzero angle α into our considerations and, as a consequence, an unrotating moment at $\alpha > 0$, arising from the normal response f_n, changes radically the properties of the mathematical impact model and at $a_\tau < 5/7$ allows the experimental data on a_τ to be satisfied under conditions both of rigid cohesion of the particles with the obstacle and of slip at the contact spot. The value α_l gives the smallest shoulder $r \sin \alpha_l$ of the force f_n (see Fig. 1), for which a particle is rotated most upon impact, and simultaneously u_2 and v_2 agree with experimental data. In the presence of slip the obtained angle α is larger than α_l.

We now pass to the case $\beta < \beta_*$, which corresponds to the inequality $\alpha_\tau > 5/7$. The assumption that upon impact the particle is in rigid cohesion with the obstacle ($\kappa \to \infty$) leads to the fact that either $\alpha_l < 0$ or with decrease in β real solutions for α_l do not exist at all. The latter means that at small β the above assumption is not fulfilled. If $\alpha_l < 0$ exists, then the normal force f_n must rotate the particle additionally to the action of f_τ, which at small angles β does not seem realistic. Thus, at $\beta < \beta_*$ it is necessary to take into consideration slip of the particle over the obstacle surface at the contact spot.

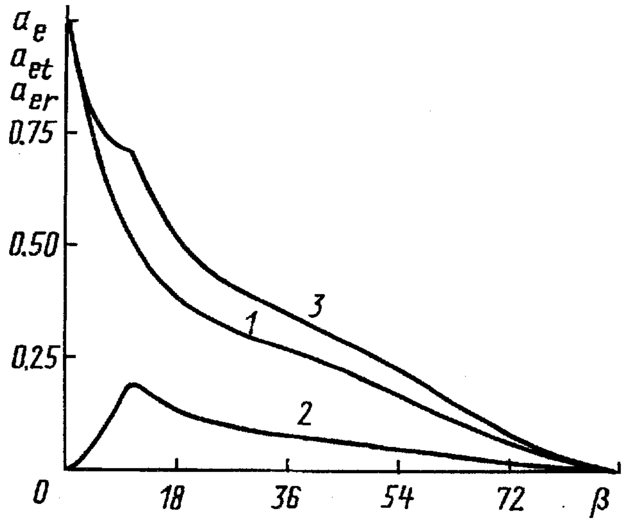

Fig. 4. Coefficient of restitution of the total kinetic energy and its components as a function of the angle of incidence β (deg) of the particle: 1) a_{et}; 2) a_{er}; 3) a_e. $V_1 = 200$ m/sec, $\omega_1 = 0$, $r = 10^{-4}$ m, the obstacle material is steel.

Since the parameters κ and α of the model are not known exactly, they may be prescribed, in accordance with the aforesaid, as follows. Starting from angles of incidence $\beta \geq \beta_*$, it is reasonable to adopt the hypothesis of rigid cohesion of a particle with the surface, i.e., to consider the limit $\kappa \to \infty$, determine α_l, and then calculate u_2 and ω_2 by formulas (7). At $\beta < \beta_*$, it is expected that the particle may slip. In this case we may assume $\alpha_l = 0$, find the corresponding value of κ from Eq. (8), and calculate u_2 and ω_2 by formulas (7). The hypothesis about the rigid cohesion of the particle with the obstacle, starting from some critical angle of incidence, agrees with [13, 14].

The assumptions formulated lead to the following relations for u_2 and ω_2:

$$u_2 = \begin{cases} u_1 a_\tau + \omega_1 r (a_\tau - 1), & \beta < \beta_*, \\ u_1 a_\tau - \dfrac{2}{2 + 5\cos\alpha_l}\omega_1 r, & \beta \geq \beta_*, \end{cases} \tag{11}$$

$$\omega_2 = \begin{cases} \dfrac{5}{2}\dfrac{u_1}{r}(a_\tau - 1) + \dfrac{5}{2}\omega_1\left(a_\tau - \dfrac{3}{5}\right), & \beta < \beta_*, \\ -\dfrac{u_1}{r}a_\tau + \dfrac{2}{2 + 5\cos\alpha_l}\omega_1, & \beta \geq \beta_*. \end{cases} \tag{12}$$

Dependences (11) and (12) may be simplified substantially, thus avoiding calculation of the function $\alpha_l(V_1, \beta)$, if we assume approximately that $\cos\alpha_l \approx 1$. In this case the error in calculating the coefficient $2/(2 + 5\cos\alpha_l)$ is 3% at $\alpha_l = 0.3$, 9% at $\alpha_l = 0.5$, and 16% at $\alpha_l = 0.7$ (the values of the angle are taken from Fig. 2). This error is obviously quite acceptable within the framework of the suggested model.

For applications it is of interest to know what portion of the kinetic energy of an incident particle converts, upon impact, to kinetic energy of the recoiling particle and how the latter is distributed between translational and rotational motion of the particle. We introduce the coefficient of restitution of the kinetic energy α_e of a particle as

$$a_e = a_{et} + a_{er},$$

$$a_{et} = \frac{m(u_2^2 + v_2^2)}{m(u_1^2 + v_1^2) + J\omega_1^2}, \quad a_{er} = \frac{J\omega_2^2}{m(u_1^2 + v_1^2) + J\omega_1^2}.$$

Results of calculations of a_{et}, a_{er}, and a_e at $\omega_1 = 0$ are shown in Fig. 4.

In conclusion, it is worth noting that the suggested simple semiempirical model of impact of a particle on a surface permits calculation of u_2 and ω_2 for both $\omega_1 = 0$ and $\omega_1 \neq 0$, which is of importance for a correct description of gas suspension flows past bodies and obstacles upon repeated recoil of particles from a surface.

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NOTATION

r, m, J, radius, mass, and moment of inertia of a particle relative to its center; V, particle velocity; u, v, projections of the velocity vector of a particle onto the tangent and normal to the surface; ω, angular velocity of particle rotation; β, angle of incidence; f, f_τ, f_n, vector of the resultant force from the obstacle on a particle upon impact and its projections onto the tangent and normal to the surface; α, angle determining the point of application of the force f; κ, resistance coefficient; T, impact time; $a_n = |v_2|/|v_1|$, $a_\tau = |u_2|/|u_1|$, coefficients of restitution of the normal and tangential velocities; a_e, coefficient of restitution of the total kinetic energy. Subscripts: 1, 2, particle parameters before and after impact.

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